# Content Descriptions

Based on the Georgia Performance Standards

# Mathematics I:
# Algebra/Geometry/Statistics



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**Introduction**

The State Board of Education is required by Georgia law (A+ Educational Reform Act of 2000, O.C.G.A. §20-2-281) to adopt End-of-Course Tests (EOCT) designed to measure student achievement in core subjects in grades nine through twelve. With educator input and State Board of Education approval, eight content areas were designated to be tested in 2001. The Georgia Performance Standards (GPS) were adopted by the State Board of Education in July 2004. Georgia educators then began the work of developing the Mathematics I EOCT based on the GPS.

**Program Purpose**

The EOCT are designed to improve student achievement by assessing student performance on the curriculum standards specific to each course tested. The results of the EOCT will be used to help make instruction more effective and to ensure that all Georgia students have access to a rigorous curriculum that meets high academic standards. Student performance on the EOCT is provided for diagnostic and remedial use. The results will also be used for student accountability and for gauging the quality of education in the state. The EOCT is the final exam for an EOCT course. The student's final grade in the course will be calculated using the course grade as 85% and the EOCT score as 15% of the final grade. The student must have a final course grade of 70 or above to pass the course and to earn credit toward graduation.

**EOCT Content Descriptions**

The EOCT Content Descriptions are provided to acquaint Georgia educators with the content coverage of the EOCT. Only the knowledge, concepts, and skills addressed in the GPS will be assessed on the EOCT. Committees of Georgia educators reviewed the curriculum and provided guidance for the assessment program.

It is important to note that some curricular standards are better suited for classroom or individual assessment rather than large-scale, paper-pencil assessment. While those curricular standards designed for classroom/individual assessment are not included in the Content Descriptions, the knowledge, concepts, and skills outlined are often required for the mastery of the standards that are assessed. Therefore, the EOCT Content Descriptions are in *no way* intended to substitute for the GPS; they are provided to help educators better understand how the curriculum will be assessed. Further, the EOCT Content Descriptions, *by no means*, suggest *when* concepts and skills should be introduced in the instructional sequence; rather, its purpose is to communicate when concepts and skills will be assessed on the EOCT. Georgia law requires educators to teach the standards set forth in the state-adopted curriculum (i.e., the GPS). The GPS is located at http://www.georgiastandards.org.

**Mathematics I Domains**

In order to provide reliable measures of student achievement, as well as to give structure to the assessment program, the content standards contained in the GPS were grouped into

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content domains. Each domain was created by combining standards that share similar content characteristics. Three domains were identified for Mathematics I:

- Algebra
  *Explore functions and solve radical, simple quadratic and rational equations; simplify and operate with radical, polynomial, and rational expressions*

- Geometry
  *Explore, understand, and use the formal language of reasoning and justification in both algebraic and geometric contexts; apply properties of polygons and determine distances and points of concurrence*

- Data Analysis and Probability
  *determine probability, find number of outcomes using both permutations and combinations; demonstrate understanding of data analysis by posing questions to be answered by collecting data; organize, represent, investigate, interpret, and make inferences from data*

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## Process Standards

The GPS in mathematics requires that content be taught in conjunction with process skills identified as the Process Standards. These process standards are necessary in order for students to master each of the mathematics content standards. Emphasis on problem solving, reasoning, representation, connections, and communication are the critical dimensions of mathematical proficiency that all students need.

The concepts and skills inherent in the Process Standards are integrated in items across the three content domains.

### Overview of the Process Standards
- Students will solve problems (using appropriate technology)
- Students will reason and evaluate mathematical arguments
- Students will communicate mathematically
- Students will make connections among mathematical ideas and to other disciplines
- Students will represent mathematics in multiple ways

### Associated GPS

MM1P1 through MM1P5 within content from MM1A1 through MM1D3

### Associated Skills
- Building new mathematical knowledge through problem solving
- Solving problems that arise in mathematics and in other contexts
- Applying and adapting a variety of appropriate strategies to solve problems
- Monitoring and reflecting on the process of mathematical problem solving
- Recognizing reasoning and proof as fundamental aspects of mathematics
- Making and investigating mathematical conjectures
- Developing and evaluating mathematical arguments and proofs
- Selecting and using various types of reasoning and methods of proof
- Organizing and consolidating mathematical thinking through communication
- Communicating mathematical thinking coherently and clearly to peers, teachers, and others
- Analyzing and evaluating mathematical thinking and strategies of others
- Using the language of mathematics to express mathematical ideas precisely
- Recognizing and using connections among mathematical ideas
- Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognizing and applying mathematics in contexts outside of mathematics
- Creating and using representations to organize, record, and communicate mathematical ideas

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- Selecting, applying, and translating among mathematical representations to solve problems
- Using representations to model and interpret physical, social, and mathematical phenomena

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**Mathematics I**
**Domain: Algebra**

**Overview of the Domain**
- Students will explore and interpret the characteristics of functions using graphs, tables, and simple algebraic techniques.
- Students simplify and operate with radical expressions, polynomials, and rational expressions.
- Students will solve radical, simple quadratic, and rational equations.

**Associated GPS Standards**

MM1A1      MM1A2      MM1A3

**Associated GPS Concepts and Skills**
Assessment of this domain will focus on the following:
- Representing functions using function notation: $f(x)$ read as $f$ of $x$
    - utilizing function notation to convey functional relationships presented in tables, graphs, and algebraic form
- Graphing and identifying graphs of basic functions, limited to $f(x) = x^n$, where $n =$ 1 to 3, $f(x) = |x|$, $f(x) = \sqrt{x}$ and $f(x) = \dfrac{1}{x}$
    - selecting a graph that matches a particular function
    - selecting a function that matches a given graph
    - understanding that graphs are geometric representations of functions
- Graphing transformations of basic functions
    - examining and identifying vertical shifts, vertical stretches, and vertical shrinks of parent functions
    - exploring and identifying reflections across the $x$- and $y$-axes of parent functions
- Investigating and explaining the characteristics of quadratic, cubic, inverse, absolute value, and square root functions, using linear functions only as a building block
    - identifying domain: the set of inputs, and range: the set of outputs
    - understanding set notation
    - exploring the zeros/solutions
    - finding $x$- and $y$-intercepts
    - determining intervals of increase and decrease
    - locating maximum and minimum values
    - explaining end behavior
- Relating to a given context the characteristics of a function
    - utilizing graphs, tables, and words to explain and predict the behavior of a function

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- – understanding the distinctions between discrete and continuous domains
- Recognizing sequences as functions with domains that are whole numbers greater than zero
  - – examining sequences given in tables, algebraically, or produced by a context and identifying the corresponding function
  - – understanding the difference between finite and infinite sequences
  - – exploring how and when to use a recursive definition for a given pattern or sequence
- Exploring rates of change
  - – comparing graphs of functions that have a constant rates of change (i.e., slope) versus variable rates of change
  - – comparing rates of change of linear, quadratic, square root, and other function families
  - – exploring average rates of change in regard to speed, cost, revenue, and other real-world applications
- Determining graphically and algebraically whether a nonlinear function has symmetry
  - – identifying if a function is even, odd, or neither
  - – interpreting if a given function has symmetry
- Understanding that any equation in $x$ can be interpreted as the equation $f(x) = g(x)$
  - – interpreting the solutions as the $x$-value(s) of the intersection points(s) of the graphs of $y_1 = f(x)$ and $y_2 = g(x)$
  - – using algebra to find the value of $x$ that makes $f(x) = g(x)$ true
  - – understanding that functions are equal if they have the same domain and rule of correspondence
- Simplifying algebraic expressions involving square roots
- Performing mathematical operations with square roots
  - – understanding when to rationalize a denominator
  - – comprehending the equivalence of a simplified square root expression and a non-simplified square root expression
- Adding, subtracting, multiplying, and dividing polynomials
- Expanding binomials, limited to the third power, using the Binomial Theorem
- Adding, subtracting, multiplying, and dividing rational algebraic expressions
- Factoring expressions involving the difference/sum of two squares, difference/sum of two cubes, and trinomials in the form $ax^2 + bx + c = 0$, where $a = 1$
  - – factoring methods limited to greatest common factor, grouping, trial and error, and special products
- Utilizing area formulas of polygons and volume models of prisms, cylinders, etc. for polynomial arithmetic
- Solving quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$, using either factorization or square roots
- Solving simple radical equations by isolating the variable and squaring both sides

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- Using technology, tables, and graphs to solve equations resulting from the investigation of $x^2 + bx + c = 0$
  - interpreting the solution of a quadratic function from a graph of the data
  - identifying and comprehending the meaning of the $x$-intercepts from a table of quadratic data
- Solving simple rational equations that result in linear equations or quadratic equations with a leading coefficient of 1

**Mathematics I**
**Domain: Geometry**

**Overview of the Domain**
- Students will investigate properties of geometric figures in the coordinate plane
- Students will understand and use the language of mathematical argument and justification
- Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons

**Associated GPS**

MM1G1        MM1G2        MM1G3

**Associated GPS Concepts and Skills**
Assessment of this domain will focus on the following:
- Determining the distance between two points on a coordinate grid
  - finding distances between two points on the same horizontal or vertical line
  - calculating the distance given two points with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ using various methods, such as the distance formula, or Pythagorean theorem
- Calculating the distance between a point and a line on a coordinate grid
  - understanding that distance between a point and a line is measured along a perpendicular.
  - exploring and understanding perpendicular lines
  - utilizing the distance formula or other methods when appropriate
- Calculating the midpoint of a segment
  - determining the midpoint of a horizontal or vertical line
  - locating the midpoint given two points on a coordinate grid with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ using various methods, such as the midpoint formula, similar triangles, averaging the endpoints, etc.
  - finding the endpoint of a line segment, given the other endpoint and the midpoint

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- Understanding the distance formula as an application of the Pythagorean theorem
  – exploring how the distance formula is derived from the Pythagorean theorem
  – finding the length of a hypotenuse or a leg of a triangle plotted on a coordinate grid
- Using the coordinate plane to investigate properties of and verify conjectures related to triangles and quadrilaterals
  – determining unknown side lengths utilizing relationship properties of side measures, slopes, diagonals, etc. of triangles and quadrilaterals
  – proving triangles and quadrilaterals similar and/or congruent utilizing side and angle theorems
  – understanding the minimal information necessary to conclude that two triangles are congruent
  – utilizing properties of parallel and perpendicular lines and angle bisectors to construct or draw the missing measure of a polygon, given a known relationship to another triangle or quadrilateral
  – utilizing the distance formula to classify figures as triangles and quadrilaterals: squares, rectangles, trapezoids, kites, parallelograms, and rhombuses
  – determining missing vertices of a triangle or quadrilateral by utilizing side and angle relationships of a given figure
- Using conjecture, inductive reasoning, deductive reasoning, counterexamples, and indirect proof as appropriate in mathematical and real-world applications
  – utilizing prior knowledge of quadrilateral relationships to prove or disprove classification of quadrilaterals
  – justifying conclusions regarding polygon relationships utilizing paragraph proofs, flow proofs, two-column proofs, or any other method that relays clear communication
- Exploring and using the relationships among conditional statements
  – determining the hypothesis and conclusion of a conditional statement, in word form or in mathematical form
  – writing the converse of a conditional statement by exchanging the hypothesis and conclusion
  – realizing that the inverse of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement
  – understanding that the contrapositive of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement and then the interchange of the hypothesis and conclusion
  – utilizing conditional statements to prove algebraic concepts, geometric concepts, and real-world concepts
- Determining the sum of interior and exterior angles in a polygon
  – utilizing angle relationships of a polygon to find a missing measure or the total interior angles measures of a specific polygon

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- – utilizing angle relationships, such as linear pairs and the exterior angle sum theorem, to determine an exterior angle of a polygon
- Understanding inequality theorems involving triangles
  - – applying the triangle inequality theorem to determine if given side lengths form a triangle.
  - – utilizing the side-angle inequality theorem to determine the largest and smallest angle or side in a triangle
  - – determining the measure of an exterior angle of a triangle given two remote interior angles, using the exterior-angle inequality theorem, linear pairs, or the sum of the angles of a triangle add to equal 180°

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- Understanding congruence postulates and theorems for triangles
  - identifying and using SSS, SAS, ASA, AAS, HL to prove/justify that given triangles are congruent through proofs including two-column, paragraph, and flow chart, or any other valid form of communication
  - understanding that SSA and AAA are not valid methods to prove triangle congruency
- Using and proving properties of and relationships among special quadrilaterals
  - parallelograms: understanding that its opposite sides are congruent, opposite angles are congruent, consecutive angles are supplementary, and its diagonals bisect each other
  - rectangle: understanding that its diagonals are congruent as well as having all properties of a parallelogram
  - rhombus: understanding that its diagonals are perpendicular and that its diagonals bisect a pair of opposite angles, as well as having all properties of a parallelogram
  - square: understanding that its diagonals are perpendicular and congruent, as well as having all properties of a parallelogram
  - isosceles trapezoid: understanding that it has only one pair of parallel sides and congruent diagonals
  - kite: understanding that it has perpendicular diagonals and that one diagonal is bisected, or opposite sides are congruent and has congruent and perpendicular diagonals, as well as having all properties of a parallelogram
  - using properties to identify and classify quadrilaterals
  - utilizing theorems to find unknown angle and side measures
- Finding and using points of concurrency, such as incenter, orthocenter, circumcenter, and centroid, in triangles
  - using bisectors, medians, and altitudes to find points of concurrency
  - locating centers of circles inscribed in or circumscribed about triangles
  - make decisions about which center best meets a given set of conditions

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**Mathematics I**
**Domain: Data Analysis and Probability**

**Overview of the Domain**
- Students will determine the number of outcomes related to a given event
- Students will use the basic laws of probability
- Students will relate samples to a population
- Students will explore variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations)

**Associated GPS**

MM1D1       MM1D2       MM1D3       MM1D4

**Associated GPS Concepts and Skills**
Assessment of this domain will focus on the following:
- Applying the addition and multiplication principles of counting
- Calculating and using simple permutations and combinations
  - integrating the multiplication principle to clarify the difference between permutations and combinations and when each is appropriate to use for a situation
  - using diagrams to justify the classification
  - utilizing permutation and combination formulas to determine the number of possible arrangements of real-world events
- Understanding when an event is mutually exclusive
  - calculating probability of mutually exclusive events using diagrams, tables, and the formula $P(A \text{ or } B) = P(A) + P(B)$
- Finding the probabilities of dependent events using diagrams, tables, and the formula $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$
  - understanding when an event is dependent
- Calculating conditional probabilities of real-world events using diagrams, tables, and the formula $P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)}$
- Using expected value to predict outcomes and make inferences
- Comparing summary statistics from one sample data distribution to another sample data distribution
  - interpreting mean, median, quartiles, and interquartile range of multiple data sets
  - understanding normal and binomial data distributions
  - describing center and variability of data distributions

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- Comparing the averages of summary statistics from a large number of samples to the corresponding population parameters
    – extracting information from two data sets, such as the set from an overall population to the set for a sample
    – interpreting histograms to make comparison of data sets
- Understanding that a random sample is used to improve the chance of selecting a representative sample
    – determining the type of sampling to be used, given a scenario, so that a survey yields results from a random population sample
    – understanding that a random sample will yield unbiased results
- Exploring variability of data by determining the mean absolute deviation: the average of the absolute values of the deviations
    – finding the greatest or least mean absolute deviation of a data set

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